# AN EXTENSION OF GENERALIZED UPPER BOUNDED TECHNIQUES I FOR STRUCTURED LINEAR PROGRAMS

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## ABSTRACT

An algorithm is developed for solving
a special structured linear program.
The particular structure studied has
a large number of blocks coupled to-
gether by a relatively few connecting
equations.  The method proposed is an
extension of [2] and, from the basis,
defines a working basis which is much
smaller in size than the original.
Two methods of updating the working
basis are proposed.

## INTRODUCTION

In this paper we propose a device for solving a large scale linear programming problem of a special configuration. The method proposed is an extension of "Generalized Upper Bounded Techniques - I" [2] to a larger class of problems.

The special configuration is the following:

$$\text{(1)} \quad \begin{cases} \boxed{A_o' x_o + A_1' x_1 + \qquad\qquad A_L' x_L} & = b_o \\[2ex] \boxed{A_1 x_1} & = b_1 \\[2ex] \boxed{A_L x_L} & = b_L \\[2ex] \gamma_o x_o + \gamma_1 x_1 + \qquad\qquad \gamma_L x_L & = z(\min) \quad . \end{cases}$$

$x$ is a vector of $N$ components which can be written $x = (x_o, x_1, \ldots, x_L)$ where $x_i$ has $n_i$ components and $N = n_o + n_1 + \ldots + n_L$.

$$A_i' \quad \text{is an} \quad m_o \times n_i \quad \text{matrix;}$$

$$A_i \quad \text{is an} \quad m_i \times n_i \quad \text{matrix} .$$

$M = m_o + m_1 + \ldots + m_L$ is the number of equations of the system which we will assume to be nonredundant and which can be written for short:

$$\text{(1')} \quad \begin{cases} \underline{Ax = b} \\ \gamma x = z(\min) \end{cases} \qquad \text{where} \quad \underline{A} \quad \text{is an} \quad M \times N \quad \text{matrix} \quad .$$

$$\boxed{\text{The basic assumption is that} \quad L \gg m_o .}$$

Although we can save an important computational work with respect to the simplex method by applying the decomposition principle [1] to Problem (1') we will, rather, taking advantage of the very special condition $L \gg m_o$, extend the device given

by Dantzig and Van Slyke in [2]: From the basis of the system a "working basis" is derived which is considerably smaller in size. Computations are done, as much as possible, on this working basis which result in a substantial savings in computation and storage. A way is given to find the next working basis from iteration to iteration.

In a first part we give definitions and theorems; in a second part we give the general method; and in a third part we give two different ways of updating.

1. <u>SOME DEFINITIONS AND THEOREMS</u> - $S_i$ will refer (depending on context) either to the set of components of $x_i$ or to the set of corresponding columns of the matrix $\underline{A}$.

Suppose we are at some stage of the process and that $\underline{B}$ is a basis for the system. Let

$$\begin{bmatrix} B_i' \\ 0 \\ B_i \\ 0 \end{bmatrix}$$

be the contribution of the set $S_i$ to the basis $\underline{B}$ .

<u>THEOREM 1</u>: At least $m_i$ variables from each set $S_i$ $(i = 1,...,L)$ are basic.

Proof: Suppose $S_p$ has less than $m_p$ basic columns. Then there exists a linear combination of rows of $B_p$ which vanishes: (i) $\lambda_p B_p = 0$ . Now we can define $\lambda = (0,...,0, \lambda_p, 0,...,0)$ such that $\lambda\underline{B} = \lambda_p B_p = 0$ , which contradicts the fact that $\underline{B}$ is nonsingular.

Remark: It follows from (i) that if $B_p$ has exactly $m_p$ basic columns, it is nonsingular.

<u>THEOREM 2</u>: The number of sets $S_i$ containing more than $m_i$ basic variables is at most $m_o$ .

Proof: By the assumption of full rank, each basis has exactly $M$ vectors. By Theorem 1, each set $S_i$ contains at least $m_i$ basic variables; this leaves $m_o$ to make up sets of more than $m_i$ basic variables.

Definition: A set $S_i$ will be said to be essential if it has more than $m_i$ basic variables; the set $S_o$ is always put in the essential class. The other sets will be called inessential.

$$\xi = \{i \mid S_i \text{ is essential}\}$$
$$\bar{\xi} = \{i \mid S_i \text{ is inessential}\} \quad .$$

II. **THE GENERAL METHOD** - The reduced system is defined by deleting the subsystems $A_i x_i = b_i$ where $S_i$ is inessential. The set of essential basic columns restricted to the reduced system is our working basis $B$ .

The value of inessential basic variables is determined from:

(1) $$B_i x_i^! = b_i$$

where $B_i$ is square and nonsingular. These smaller systems are readily solved.

THEOREM 3: The working basis is a basis for the reduced system.

Proof: The number of equations in the reduced system equals the number of variables in the working basis since for each inessential set we remove exactly $m_i$ equations and $m_i$ basic columns from the square basis $\underline{B}$ . The columns of the working basis are linearly independent since they differ from the columns of the basis $\underline{B}$ by a bunch of zeros in the inessential part, and the columns of $\underline{B}$ are linearly independent.

The size of the working basis $B$ is $m_o + \sum_{i \in \xi} m_i$ , considerably smaller than that of $\underline{B}$ . Note that the size of $B$ changes from step to step, and that even the number of blocks in $B$ (which is bounded by $m_o$) changes from step to step.

Suppose we have a basis $\underline{B}$ , the columns of which will be denoted $\underline{B}^j$ (j=1,...

Let $B = [B^{h_i}, i = 1,...,|B|]$ be the corresponding working basis.

The problem is:

    a) To determine which column enters the basis;

    b) To determine which column is dropped from the basis.

  1) Let

$$\underline{\pi} = (\pi_o, \pi_i, \ldots, \pi_L) \quad \text{be the price vector corresponding to } \underline{B} ;$$

$$\pi = (\pi_o, \pi_1 \mid i \in \xi) \quad \text{be the price vector corresponding to } B ;$$

$$\gamma = (\gamma_o, \gamma_1, \ldots, \gamma_L) \quad \text{be the cost vector;}$$

$$\underline{c} = (c_o, c_1, \ldots, c_L) \quad \text{be the cost vector associated with } \underline{B} ;$$

$$c = (c_i \mid i \in \xi) \quad \text{be the cost vector associated with } B .$$

The equation

(2) $$\underline{\pi} \, \underline{B} = \underline{c}$$

defines the price vector.  The point is that this equation can be written

(2') $$\pi B = c \quad ;$$

(2'') $$\pi_o B_i^! + \pi_i B_i = c_i \qquad i \in \bar{\xi} \quad .$$

We will later give different ways of solving (2'); (2') being solved, substituting $\pi_o$ in (2'') gives $\pi_i$ in an easy way.

Let $A_\sigma^s$ and $A'^s_\sigma$ denote the $s^{th}$ column of matrices $A_\sigma$ and $A'_\sigma$ respectively.

$$\underline{A}_\sigma^s = \begin{pmatrix} A'^s_\sigma \\ 0 \\ A^s_\sigma \\ 0 \end{pmatrix} \qquad \text{is the corresponding column of } \underline{A} \quad .$$

The column to enter the basis will be given by the usual criterion:

$$\bar{\gamma}_\sigma^s = \gamma_\sigma^s - \underline{\pi} \, \underline{A}_\sigma^s = \underset{\substack{0 \le \tau \le L \\ 1 \le j \le n_\tau}}{\text{Min}} \; (\gamma_\tau^j - \underline{\pi} \, \underline{A}_\tau^j) \qquad .$$

If $\bar{\gamma}_\sigma^s \ge 0$, we have found an optimal basic solution; if not, we bring $\underline{A}_\sigma^s$ into

the basis.

2) $\underline{A}_\sigma^s \in S_\sigma$ . If $S_\sigma$ is essential we have not to worry about the inessential

part of the problem which remains unchanged. We simply pivot in the reduced system

with the usual criterion for picking the column to drop. We then have to update

the right-hand side, and the entering column is the reduced system. That is, to

solve

(3) $\qquad B \, \tilde{A}_\sigma^s = A_\sigma^s$ ;

(4) $\qquad B \, \tilde{b} = b$ .

We postpone to a later part the discussion of how (3) and (4) are actually

solved. Note that in this case, since $B$ is assumed to be of a reasonable size,
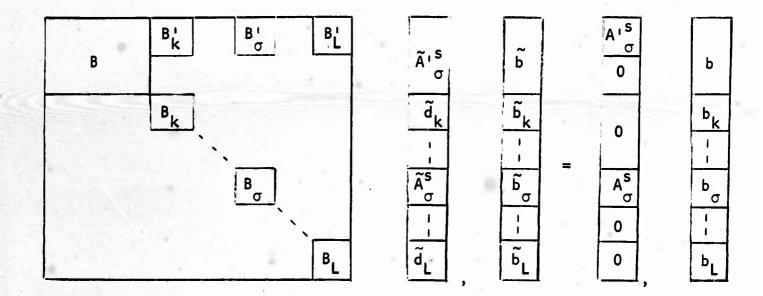
the updating is simple.

If $S_\sigma$ is not essential we have to solve, instead of (3) and (4), the more

complicated systems:

(3') $\qquad \underline{B} \, \underline{\tilde{A}}_\sigma^s = \underline{A}_\sigma^s$ ;

(4') $\qquad \underline{B} \, \underline{\tilde{b}} = \underline{b}$ .

We will now prove that solving (3') and (4') is, in fact, equivalent to solving (3)

and (4).

Let $\tilde{A}'^s_\sigma$ , $\tilde{b}$ denote the part of the updated columns which correspond to the

working basis.

This can be written:

$$(5) \quad \begin{cases} B_k & \tilde{d}_k = 0 \\ & \vdots \\ & = 0 \\ B'_\sigma & \tilde{A}^s_\sigma = A^s_\sigma \\ & = 0 \\ B_L & \tilde{d}_L = 0 \end{cases} \qquad (6) \quad \begin{cases} B_k & \tilde{b}_k = b_k \\ & \vdots \\ B'_\sigma & \tilde{b}_\sigma = b_\sigma \\ & \vdots \\ B_L & \tilde{b}_L = b_L \end{cases}$$

$$(3'') \quad B\tilde{A}'^s_\sigma = \begin{bmatrix} A'^s_\sigma \\ 0 \end{bmatrix} - \begin{bmatrix} B'_\sigma \\ 0 \end{bmatrix} \tilde{A}^s_\sigma \ ; \qquad (4'') \quad B\tilde{b} = b - \sum_{i \in \bar{\xi}} \begin{bmatrix} B'_i \\ 0 \end{bmatrix} \tilde{b}_i \ .$$

And we see that if we suppose the systems in (5) and (6) are of very low order and that their solution is readily at hand, we have only to solve (3'') and (4''), which are exactly of the same type as (3) and (4).

By the usual simplex criterion we now know which column is to be dropped. Let us suppose that the column to be dropped belongs to the set $S_\tau$. Two cases can occur:

      a) $S_\tau$ is inessential, so $S_\tau$ can only be $S_\sigma$. Then the pivoting is only done in the inessential sub-basis $B_\sigma$, and the working basis does not change.

b) $S_\tau$ is essential, so the method of pivoting is as follows:

    <u>Step I</u>  -  Introduce $S_\sigma$ in the essential set (the size of the working basis is increased by $m_\sigma$) .

    <u>Step II</u>  -  Pivot in the new reduced system.

    <u>Step III</u> -  If the number of variables in $S_\tau$ is now $m_\tau$ ,[†] make $S_\tau$ inessential (the size of the working basis is de-creased by $m_\tau$) .

The algebraic work involved in Equations (2'), (3) and (4) (or (3") and (4") ) can be done in essentially two ways. The first is the Revised Simplex Method; the justification of using it is that the working basis will remain of a reasonable size. The second is to solve the system, instead of inverting the matrix $B$ , by a trian-gularization process which is inspired of [3].

## III. <u>UPDATING PROCESS</u> -

A. <u>Revised Simplex Method</u>: Suppose we have an inverse of the working basis $B^{-1}$; we want to find $B^{*-1}$ where $B^*$ is the basis for the next iteration. If $S_\sigma$ is essential, we pivot in the reduced system and simply apply the revised simplex algorithm in the reduced system. If $S_\sigma$ and $S_\tau$ are inessential, the working basis does not change. If $S_\sigma$ is inessential and $S_\tau$ is essential, we will de-scribe the three steps outlined above.

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[†] If the essential set $S_\tau$ had more than $m_{\tau+1}$ basic columns beforehand, this will not be the case.

**Step 1.** Define

$$\tilde{B} = \begin{bmatrix} B & B'_\sigma \\ 0 & B_\sigma \end{bmatrix} \qquad Q = \begin{bmatrix} B'_\sigma \\ \phantom{0} \end{bmatrix} \qquad ;$$
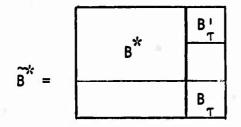
then it is easy to see that:

$$\tilde{B}^{-1} = \begin{bmatrix} B^{-1} & Q' \\ 0 & B_\sigma^{-1} \end{bmatrix} \qquad \text{with} \quad Q' = B^{-1}QB_\sigma^{-1} \qquad .$$

**Step 2.** Now we are as in the first case and we use the modified simplex method to get $\tilde{B}^{*-1}$ (where $\tilde{B}^*$ is obtained from $\tilde{B}$ by pivoting in the reduced system).

**Step 3.** If $B_\tau$ is now square, we can rearrange the equations and variables in $\tilde{B}^*$ so that it looks like this:

$$\tilde{B}^* = \begin{bmatrix} B^* & B'_\tau \\ & B_\tau \end{bmatrix} \qquad \tilde{B}^{*-1} = \begin{bmatrix} B^{*-1} & Q'_\tau \\ & B_\tau^{-1} \end{bmatrix} \qquad .$$

$B^*$ and $B^{*-1}$ are now obtained by dropping the column and rows corresponding to $B_\tau$ .

B. <u>Compact Basis Triangularization</u>:  Any equations of the type  $Bx = b$  or $\pi B = c$  where  B  is an  m x m  nonsingular matrix can be solved by triangularizing B  (see [3]).  It has been pointed out in [3] that triangularizing may be an efficient method in cases where  B  has a special structure (such as block angular).

Triangularizing is equivalent to pre-multiplying  B  by a set of elementary matrices  $E_i$

$$T = E_1, \ E_2, \ldots, E_m B$$

where, in the notation of [3],

T  is the triangularized form of  B ;

$E_1, \ldots, E_m$  is the compact  E-structure.

To solve  $\pi B = c$  (Equations (2') and (2'') ), we solve

$$\pi^* T = c$$

and

$$\pi = E_1, \ E_2, \ldots, E_m \pi^* \quad .$$

To solve  $Bx = b$  (Equations (3),(4),(5),(6) ), we find

$$b^* = E_1, \ E_2, \ldots, E_m b$$

and solve

$$Tx = b^* \quad .$$

Having determined  $S_\sigma$  and  $S_\tau$ , we now show the steps in pivoting.

If  $S_\sigma$  is essential, we pivot in the reduced working basis.  If  $S_\sigma$  and $S_\tau$  are inessential, we pivot in the smaller basis  $B_\sigma$ .  If  $S_\sigma$  is inessential and  $S_\tau$  essential, then we show the three steps.

Let  $T_\sigma$  be the triangularized form of  $B_\sigma$ ,  $E_\sigma$  its compact  E-structure, and  J  the permutation matrix used to triangularize  B .  Let  T  be the

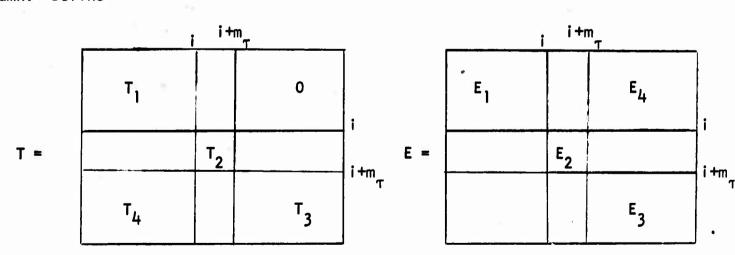triangularized form of $B$, and $E$ its compact $E$-structure.

$\underline{\text{Step 1.}}$ Introducing $S_\sigma$ into the essential class, we modify the working basis by introducing $B_\sigma$ into it. We get[†]

New T =

| $T_\sigma$ | 0 |
|---|---|
| EJQ | T |

Q =

| $B'_\sigma$ |
|---|
| 0 |

New E =

| $E_\sigma$ | 0 |
|---|---|
| 0 | E |

$\underline{\text{Step 2.}}$ Apply the pivot operation in the new $B$.

$\underline{\text{Step 3.}}$ Assume $\xi_\tau$ has become inessential; to remove this block from the working basis, assume the columns of $S_\tau$ in $B$ lie between $i^{th}$ and $i+m_\tau{}^{th}$ column. Define

$T =$

| $T_1$ | | 0 |
|---|---|---|
| | $T_2$ | |
| $T_4$ | | $T_3$ |

$E =$

| $E_1$ | | $E_4$ |
|---|---|---|
| | $E_2$ | |
| | | $E_3$ |

Then

---

[†] Note: If we introduce the inessential set $S_\sigma$ at the top LHS of the $T$ and $E$, we always obtain the new $E$ structure in this manner.

triangularized form of $B$ , and $E$ its compact E-structure.

**Step 1.**     Introducing $S_\sigma$ into the essential class, we modify the working basis by introducing $B_\sigma$ into it. We get[†]

New T =

$$\begin{array}{|c|c|}
\hline
T_\sigma & 0 \\
\hline
EJQ & T \\
\hline
\end{array}$$

Q =

$$\begin{array}{|c|}
\hline
B'_\sigma \\
\hline
0 \\
\hline
\end{array}$$

New E =

$$\begin{array}{|c|c|}
\hline
E_\sigma & 0 \\
\hline
0 & E \\
\hline
\end{array}$$

**Step 2.**     Apply the pivot operation in the new $B$ .

**Step 3.**     Assume $\xi_\tau$ has become inessential; to remove this block from the working basis, assume the columns of $S_\tau$ in $B$ lie between $i^{th}$ and $i+m_\tau{}^{th}$ column.   Define

T =

$$\begin{array}{|c|c|c|}
\hline
T_1 & & 0 \\
\hline
 & T_2 & \\
\hline
T_4 & & T_3 \\
\hline
\end{array}$$

E =

$$\begin{array}{|c|c|c|}
\hline
E_1 & & E_4 \\
\hline
 & E_2 & \\
\hline
 & & E_3 \\
\hline
\end{array}$$

Then

---

[†]   Note:  If we introduce the inessential set $S_\sigma$ at the top LHS of the  $T$  and

$E$ , we always obtain the new  $E$  structure in this manner.

$$\text{new } T = \begin{array}{|c|c|} \hline T_1 & 0 \\ \hline T_4 & T_3 \\ \hline \end{array} \qquad \text{new } E = \begin{array}{|c|c|} \hline E_1 & E_4 \\ \hline & E_3 \\ \hline \end{array}$$

and $\qquad T_\tau = T_2 \qquad\qquad E_\tau = E_2$ .

In conclusion, it seems that Method A may be preferred if the working basis is of small size and if the process of the revised simplex algorithm on it is not too long, as Method B will be preferred if the working basis itself is of large dimension.

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         Australia (March 1963).

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**13. ABSTRACT**

An algorithm is developed for solving a special structured linear program. The particular structure studied has a large number of blocks coupled together by a relatively few connecting equations. The method proposed is an extension of "Generalized Upper Bounding Techniques for Linear Programming-I," by G. B. Dantzig and R. M. Van Slyke (March 1965) and, from the basis, defines a working basis which is much smaller in size than the original. Two methods of updating the working basis are proposed.

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